Basophils and amplitude amplification

I give here a simple example of the mechanism which, according to my conjecture, amplifies the signal induced by the antibodys amplitude in the sample. The example is based on the techniques and ideas of amplitude amplification, first introduced in Grovers algorithm for database search and then developed by Brassard and others (see [1] for a good introduction).

Let the state-vector of the unperturbed state of the basophils sensors be $|0\rangle$ and consider the perturbation induced by the antibodys presence in the sample so that the excited state of the sensors is $|c\rangle = m_\lambda|0\rangle + \lambda|1\rangle$ where λ is the very small amplitude associated to the probability that an antibody molecule may be localized in the sample and m_λ is chosen so as to keep $|c\rangle$ normalized. Without loss of generality we can assume that $|0\rangle$ and $|1\rangle$ are orthogonal. Consider now the space S spanned by $|0\rangle$ and $|1\rangle$, which we identify as the sensors' state space. We assume that $|0\rangle$ and $|1\rangle$ are eigenvectors of an appropriate operator K, whose measurement corresponds either to degranulation or to its absence, depending on whether it yields $|1\rangle$ or $|0\rangle$ respectively. In the terminology of [1] we call $|1\rangle$ a "good" state. A measurement of $K|c\rangle$ in S will yield $|1\rangle$ with probability $|\lambda|^2$, i.e. with the probability that an antibody molecule may be localized in the sample by an appropriate measurement. We assume that the evolution of the sensorsstate space is governed by an operator Z_c which we describe and which depends on the sensors state. For any state v in S let W^v be the unitary rotation such that $W^v|0\rangle = |v\rangle$ and consider the the operator Z defined as

$$Z_v = Z_v(\phi, \psi) = -W_v S_0^\psi W_v^{-1} S_1^\phi$$

where S_k^ρ is defined as in [1] as

$$S_v^\rho|s\rangle \mapsto \rho|s\rangle \quad if \quad v = s$$

$$S_v^\rho|s\rangle \mapsto |s\rangle \; if \quad v \neq s$$

for any complex complex number ρ of unit norm. The operator Z_c is at the core of the amplitude amplification process. It is an instance of the operator Q in [1], obtained by taking W_c as A. We apply now Theorem 1 on Amplitude Amplification from [1]. We obtain that for $Z_c = Z_c(-1,-1)$ for all $j \geq 0$

$$Z_c^j|c\rangle = l_j m_\lambda|0\rangle + k_j\lambda|1\rangle$$

where

$$k_j = \frac{1}{|\lambda|} sin((2j+1)\theta) \qquad l_j = \frac{1}{\sqrt{1-|\lambda|^4}} cos((2j+1)\theta)$$

and θ is defined so that $sin^2(\theta) = |\lambda|^2$ and $0 \leq \theta \leq \pi/2$. So if we (or the basophil) apply the operator Z_c^j to the sensors' state vector, the probability for a measurement of its state vector to yield $|1\rangle$ is given by $|\lambda|^2 k_j^2 = sin^2((2m+1)\theta)$. Note that if there is no antibody amplitude in the sample, i.e. if $|\lambda|^2 = 0$ and $|c\rangle = |0\rangle$ the probability to measure $|1\rangle$ will stay null. We may also speculate that it is unlikely that the basophil may choose j in an optimal way so as to always boost the perturbations amplitude. Different dilutions will induce the same perturbation $|1\rangle$ but with different values of θ. The antibody "expects" $|1\rangle$ and is tuned on it but no a "a priori" knowledge of θ should be assumed, since the basophil reacts to the antibodys presence at different dilutions. However, even without knowing θ a priori, the algorithm may boost the chance to detect $|1\rangle$ substantially, based only on an assumption on the upper bound of the dilution range where the detection mechanism is effective. Assuming that $(2j+1)\theta \gg 2\pi$ on the dilution range, we have that the probabilty of a measurement of $Z_c^j|c\rangle$ to yield $|1\rangle$ is about 0.5 i.e. the average value of $sin^2((2m+1)\theta)$ taken on the dilution range.

Note that at certain dilutions there will be no amplitude amplification, while at other dilutions the amplification will be substantial, bringing the detection probability close to 1.

Under the assumptiom that the mechanisms implemented by the basophils relies on an operator of type Z^j, the above dependence of the amplitude boosting algorithm on the dilution may be the source the irregular degranulation patterns that were construed as the "unknown variation source" in Hirst et al. and interpreted in terms of "dilution waves" in Davenas et al..

This completes the example.

That the above model is highly speculative, to put it mildly, is clear even to its author. It may however have an heuristic value since it shows how the actual amplification process is induced by the modified Hamiltonian and perturbation theory may give us some qualitative and even quantitave indication about the conditions that may trigger it.

Assuming that Hamiltonian governing the basophlis sensors is not degenerate we may consider its energy eigenvalues E_0, E_1 where we associate E_0 with the unperturbed stationary state $|0\rangle$ where no degranulation takes place and where the eigenvector $|1\rangle$ associated to the eigenvalue E_1 with corresponds to degranulation. Let H be the Hamiltonian of the unperturbed stationary system. We investigate the dynamics induced by an Hamiltonian W acting in a time interval $[0, T]$ such that a $H + \lambda W$ induces a non-null transition probablity from $|0\rangle$ to $|1\rangle$. Consider W such that

$$\langle 0|W|0\rangle = 0 \qquad \langle 1|W|0\rangle = \hbar\omega_0 \qquad \langle 1|W|1\rangle = \hbar\omega_{01}$$

where $\omega_{01} \neq 0$ is the non-null Bohr frequency enabling the resonant behavoiur of the device. Considering the explicit solution of the Schroedinger equation

$$|\psi(t)\rangle = C_1(t)e^{-iE_1t/\hbar}|0\rangle + C_2(t)e^{-iE_2t/\hbar}|1\rangle$$

and substituting in

$$i\hbar\frac{\partial\psi(t)}{\partial t} = (H + \lambda W)|\psi(t)\rangle$$

we obtain after some manipulation that the transition probability W_{01} for $t \geq T$ is given by

$$W_{01} = sin^2(\lambda\omega_0 t)$$

so that so that the probability to measure $|1\rangle$ and hence to detect the perturbation W is one for times $T = \pm\pi/2\omega_0 + \pi k/\omega_0$ for $k \in N$. Again, if there is no perturbation, the state of the sensors will not budge from $|0\rangle$. As a next step one can try to get some estimate of the values involved by fitting the experimental data at our disposal into the formulas that we have obtained.

[1] Gilles Brassard, Peter Hoyer and Alain Tapp "*Quantum Counting*" Preprint. At http://www.imada.ou.dk/ u2pi/papers/BHT98b.abstract.html.

[2] Walter Greiner "*Quantum Mechanics - An Introduction*" Springer Verlag, 1989.

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